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COMMENT

Breakdown of dynamic scaling in the 2D diluted Potts model

J M Nunes da Silva and E J S Lage

Laboratório de Física, Faculdade de Ciências, Universidade do Porto, Portugal

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Abstract. The recently found breakdown of dynamic scaling of the kinetic Ising model (at the percolation threshold) is here shown to be exhibited, also, by the single-spin flip, dynamical q-state Potts model. We find $A(q)/A(2)=2(q-1)/q$, in good agreement with very recent Monte Carlo simulations.

The dynamic scaling hypothesis for the characteristic relaxation time, near the bicritical point of a diluted system, should be

$$\tau_c = \xi^z f(\xi_T/\xi_p) \tag{1}$$

where $\xi=\xi(T,p)$ is the correlation length and $\xi_T=\xi(T,p_c)$, $\xi_p=\xi(0,p)$ and z is the critical dynamical exponent (Harris 1984, Kumar 1984). However, very recently it has been found that this hypothesis breaks down at the percolation threshold of a diluted Ising system (Harris and Stinchcombe 1986, Stinchcombe 1985, Jain 1986). Instead of equation (1) one finds that (for $p=p_c$)z is a function of temperature:

$$z = A\ln\xi_T + B. \tag{2}$$

Recent Monte Carlo simulations (Jain 1986, Jain *et al* 1986) for the bond diluted $q=2$ and $q=3$ Potts model support the idea that A and B in (2) are q dependent. In particular, it is found that $A=0.78\pm0.15$ for $q=3$ and $A=0.51\pm0.05$ for $q=2$.

Here we follow closely the method which one of us (Lage 1986) has applied to the Ising model on a quadratic lattice (lattice constant $\boldsymbol{a}$). This method has proved satisfactory for obtaining equation (2) and we simply generalise it to the q-state Potts model. We take single-spin rates defined by (Lage 1985)

$$w(\alpha_i \to \alpha_i') = \frac{1}{\tau(a)}\exp\left(-\sum_j K_{ij}(\delta_{\alpha_i,\alpha_j} - q^{-1})\right).$$

The real space renormalisation group method (with lattice scaling factor $b=\sqrt{2}$) is now applied to the master equation using the method developed by Achiam and Kosterlitz (1978). It is found (see Lage (1986) for details) that the most divergent contribution to the characteristic time at the percolation threshold renormalises into

$$\tau(ba) = 2\tau(a)[p_c(1-p_c)^2]^4\left(\frac{\xi^{(q-1)/\nu_p}}{q}\right)^{4/q}$$

where ν_p is the pure percolation exponent. For $q=2$, this reduces to the result found for the Ising model.

Following Stinchcombe (1985) we obtain

$$A^{-1} = \frac{\nu_p}{2} \frac{q}{q-1} \ln b$$

and

$$B = \frac{2}{\nu_p} \frac{q-1}{q} + 4\left(\ln \frac{2p_c(1-p_c)^2}{q^{1/q}} \right) (\ln b)^{-1}.$$

Although the numerical values of these coefficients are not in good agreement with MC simulations due to the approximations used, we nevertheless also confirm the q dependence of A (increase of about 30% from $q = 2$ to $q = 3$). We hope in the near future to improve our method and obtain better numerical results.

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